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Quantum Entanglement Dynamics of Two Atoms in Quantum Light Sources*

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Abstract *Quantum entanglement dynamics of two Tavis–Cummings atoms interacting with the quantum light sources in a cavity is investigated. The results show the phenomenon that the concurrence disappears abruptly in a finite time, which depends on the initial atomic states and the properties of squeezed states. We find that there are two decoherence-free states in squeezed vacuum fields: one is the singlet state, and the other entangled state is the state that combines both excited states and ground states with a relative phase being equal to the phase of the squeezed state.*

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1 Introduction

Quantum entanglement (QE) plays a leading role in quantum optics and the fundamentals of quantum mechanics and it is known as an important resource for quantum information science (QIS),[1–2] such as quantum teleportation, quantum cryptography, and quantum dense coding, etc. For instance, entanglement-assisted quantum communication can enlarge the capacity[3] and enhance the efficiency of quantum channels.[4] So, it is of great importance to study entanglement dynamics in diverse scenarios.

In 2004, Yu and Eberly showed that two initially entangled qubits without interaction between each other could be later suddenly disentangled completely.[5] The phenomenon, named the entanglement sudden death (ESD) afterwards, is distinctly different from the behavior of local decoherence process, which takes an infinite time evolution under the influence of vacuum fluctuations. Many researchers have devoted themselves to the realm both in theory[6–12] and in experiment[13–16] since the first confirmation of the ESD by Almeida and his collaborators in 2007.[17] Although people have already tried to seek the essence of ESD, its mechanism is still unclear.[18]

Cavity quantum electrodynamics (CQED), studying the interaction between light and atoms (or ions, atomic ensembles etc.) in a confined space, is thought to be a potential candidate for the demonstration of quantum state engineering.[19] ESD was also well studied in the CQED context via J–C models[20–25] and the extensive J–C models (T–C models).[26–32] In those proposals, the bath is usually taken as the vacuum reservoir or classical fields (such as thermal fields). The ESD effect does appear in some cases but we are still lack of the general conditions for the ESD occurrence. For example, the result that the presence of both atoms in excited states is a necessary condition for ESD in vacuum reservoir with J–C model, but this result is not suitable for the broader quantum states.[31]

In this paper, we have investigated the entanglement behavior of two Tavis–Cummings atoms interacting with the squeezed vacuum states (SVS), a typical quantum light source,[33] for various initial atomic states. Our numerical results show that the atomic concurrence is closely related to the initial atomic state and the properties of the light sources, such as the phase and the squeezing factor and the period of the sudden death depends on the initial atomic states. In addition, we have also shown that there exist two decoherence-free states in SVS including the singlet state and the state with the combination of both excited states and ground states, which is different from early result obtained in the thermal fields.[34]

2 Model and Measurement for Two-Atom QE

The Tavis–Cummings model for two identical two-level atoms interacting with a single cavity mode (indicated as cavity C) is considered and shown in Fig. 1, where the two atoms with the ground state $|g\rangle$ and the excited state $|e\rangle$ are labeled by the subscripts A and B. For simplicity, we assume that there is no direct interaction between the two atoms and they are both resonant with the cavity mode with the same, real coupling rate g. In this situation, the interaction Hamiltonian for the total system, in the dipole approximation and the rotating-wave approximation, is[35] ($\hbar = 1$)

$$H_I = g \sum_{i=A,B} (a^\dagger \sigma_i^- + a\sigma_i^+)\,, \tag{1}$$

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where $a(a^\dagger)$ represents the annihilation (creation) operator for the cavity field, $\sigma^- = |g\rangle\langle e|$ and $\sigma^+ = |e\rangle\langle g|$ are the raising and lowing operators, respectively.

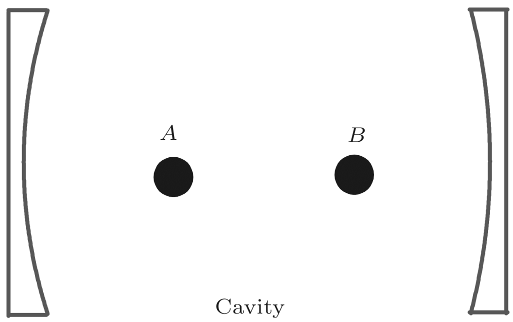

Fig. 1 Schematic diagram of the Tavis–Cummings model. Two identical atoms A and B couple to a single-mode cavity field. There is no interaction between these two atoms initially.

If we suppose that the two atoms and the cavity field have no interaction initially, then the density operator for the initial system state can be expressed as

$$\rho(0) = \rho_{AB}(0) \otimes \rho_C(0)\,, \tag{2}$$

with $\rho_{AB}(0)$ and $\rho_C(0)$ representing the initial atomic and cavity's density matrices, respectively. Under the action of the interaction Hamiltonian (1), the system state, at time t, evolves to

$$\rho(t) = \mathrm{e}^{-\mathrm{i}Ht}\rho_{AB}(0) \otimes \rho_C(0)\,\mathrm{e}^{\mathrm{i}Ht}\,. \tag{3}$$

By tracing the system's density matrix over the cavity field, the density matrix for the two atoms is

$$\rho_{AB}(t) = \mathrm{Tr}_C\,\rho(t)\,. \tag{4}$$

Similar to the previous discussions, here we also use the concurrence to quantify the degree of entanglement for $\rho_{AB}(t)$, expressed as[36]

$$C(\rho_{AB}) = \max\{0,\, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}\,, \tag{5}$$

with λ_i ($i = 1, 2, 3, 4$) being the square root of the eigenvalues of the matrix $\rho_{AB}(\sigma_y \otimes \rho_y)\rho^*_{AB}(\sigma_y \otimes \rho_y)$ in decreasing order and ρ^*_{AB} the complex conjugation of ρ_{AB} in the standard basis and σ_y is a Pauli matrix expressed as $\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$. The concurrence $C(\rho_{AB})$ varies from 0 to 1, corresponding to disentanglement and the maximum entanglement, respectively. Besides, it is also proved that the concurrence for the X-class state[8]

$$\rho_{AB} = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \tag{6}$$

with a, b, c, d being positive and satisfying $a+b+c+d = 1$, and w, z being complex quantities, can be simplified as

$$C(\rho_{AB}) = 2\max\{0,\, |z| - \sqrt{ad},\, |w| - \sqrt{bc}\}\,. \tag{7}$$

3 QE Dynamics of Two Atoms in Squeezed Vacuum States

We now discuss the QE dynamics for two atoms initially in the SVS, expanded in the Fock-state basis[33]

$$\begin{aligned} |\psi_{\mathrm{SVS}}\rangle &= (1-\xi^2)^{1/4} \sum_{n=0}^{\infty} \frac{(-\xi\,\mathrm{e}^{\mathrm{i}\theta})^n \sqrt{(2n)!}}{2^n n!} |2n\rangle \\ &= (1-\xi^2)^{1/4} \sum_{n=0}^{\infty} \frac{\chi_n \cdot (-\xi\,\mathrm{e}^{\mathrm{i}\theta}/2)^{n/2} \sqrt{n!}}{(\chi_n \cdot n/2)!} |n\rangle, \end{aligned} \tag{8}$$

where $\chi_n = (1 + (-1)^n)/2$, $\xi = \tanh r$ with r representing the squeezing factor. Its mean photon number is $\bar{n}_{\mathrm{SVS}} = \sinh^2 r = \xi^2/(1-\xi^2)$. We suppose the two atoms are initially prepared either in the ψ-type Bell state

$$|\psi_{AB}\rangle = \cos\alpha|eg\rangle + \sin\alpha|ge\rangle\,, \tag{9a}$$

or in the Φ-type Bell state

$$|\Phi_{AB}\rangle = \cos\alpha|ee\rangle + \sin\alpha|gg\rangle\,. \tag{9b}$$

For clarity, the system evolution and the atomic concurrence for ψ-type Bell states and Subsec. 3.2 for Φ-type Bell states is described in Subsecs. 3.1 and 3.2, respectively.

3.1 *System Evolution and Atomic Concurrence for ψ-type Bell States*

Under the action of the interaction Hamiltonian (1), the system's density matrix, at the interaction time t, evolves from

$$\rho(0) = |\psi_{AB}\rangle\langle\psi_{AB}| \otimes |\psi_{\mathrm{SVS}}\rangle\langle\psi_{\mathrm{SVS}}|\,, \tag{10}$$

to

$$\begin{aligned} \rho(t) = \mathrm{e}^{-\mathrm{i}Ht}\rho(0)\,\mathrm{e}^{\mathrm{i}Ht} &= (1-\xi^2)^{1/2} \sum_{m,n=0}^{\infty} \frac{\chi_m\chi_n \cdot (-\xi/2)^{(m+n)/2}\,\mathrm{e}^{\mathrm{i}(m-n)\theta/2}\sqrt{m!n!}}{(\chi_m \cdot m/2)!(\chi_n \cdot n/2)!} \\ &\quad\times (C_{eem}|ee\rangle|m-1\rangle + C_{egm}|eg\rangle|m\rangle + C_{gem}|ge\rangle|m\rangle + C_{ggm}|gg\rangle|m+1\rangle) \\ &\quad\times (C^*_{een}\langle ee|\langle n-1| + C^*_{egn}\langle eg|\langle n| + C^*_{gen}\langle ge|\langle n| + C^*_{ggn}\langle gg|\langle n+1|)\,, \end{aligned} \tag{11}$$

with

$$C_{ee,n} = \frac{-\mathrm{i}\sin(\alpha+\pi/4)\sqrt{n}\sin(gt\sqrt{2(2n+1)})}{\sqrt{2n+1}}, \quad C_{eg,n} = \frac{\sin(\alpha+\pi/4)}{\sqrt{2}}\cos(gt\sqrt{2(2n+1)}) + \frac{\sin(\alpha-\pi/4)}{\sqrt{2}},$$

$$C_{ge,n} = \frac{\sin(\alpha+\pi/4)}{\sqrt{2}}\cos(gt\sqrt{2(2n+1)}) - \frac{\sin(\alpha-\pi/4)}{\sqrt{2}}, \quad C_{gg,n} = \frac{-\mathrm{i}\sin(\alpha+\pi/4)\sqrt{n+1}\sin(gt\sqrt{2(2n+1)})}{\sqrt{2n+1}}, \tag{12}$$

and $C_{x,n}^*$ $(x = ee, eg, ge, gg)$ being the complex conjugate of $C_{x,n}$. By tracing over the cavity field, the density matrix for the atomic system, in the basis of $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, can be described in the form of Eq. (6) with the matrix elements:

$$\begin{aligned}
a &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}|c_{ee,n}|^2,\\
b &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}|c_{eg,n}|^2,\\
c &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}|c_{ge,n}|^2,\\
d &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}|c_{gg,n}|^2,\\
z &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}c_{eg,n}c_{ge,n}^*,\\
w &= \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^{n+1}\sqrt{n!(n+2)!}}{(\chi_n\cdot n/2)\,!(\chi_{n+2}\cdot(n+2)/2)\,!}\\
&\quad\times\,\mathrm{e}^{\mathrm{i}\theta}c_{ee,n+2}c_{gg,n}^*\,. \qquad (13)
\end{aligned}$$

In order to show the time evolution of the atomic concurrence, we substitute Eqs. (13) to (7). It is clearly seen that the concurrence is independent on the SVS phase θ. The concurrence C_{AB} as the function of scale time gt/π and α is depicted in Fig. 2, where ξ is chosen to be (a) $\sqrt{0.2}$, (b) $\sqrt{0.5}$, (c) $\sqrt{0.9}$, corresponding to the mean photon numbers $\bar{n}_{\rm SVS}$ = (a) 0.25, (b) 1, (c) 9, respectively, and α is confined within the range $[0,\pi]$. The numerical results show that the concurrence is periodically oscillating. It disappears abruptly in a finite time (ESD) when $\alpha\in(0,\pi/2)$ and the larger mean photon number of the SVS is, the more obvious the ESD appears. Besides, for the singlet state $|\psi_{AB}(3\pi/4)\rangle = (-|eg\rangle|ge\rangle)/\sqrt{2}$, the concurrence does not decay in the squeezed vacuum state, which implies that it is a decoherence-free states (DFS).[34]

3.2 *System Evolution and Atomic Concurrence for Φ-type Bell State*

Let us consider the system evolution for the initial Φ-type Bell states described in Eq. (9b). In this case, the initial system is

$$|\rho(0)\rangle = |\Phi_{AB}\rangle\langle\Phi_{AB}|\otimes|\psi_{\rm SVS}\rangle\langle\psi_{\rm SVS}|\,. \qquad (14)$$

After the interaction time t, the system will be

$$\begin{aligned}
|\rho(t)\rangle =&(1-\xi^2)^{1/2}\sum_{m,n=0}^{\infty}\frac{\chi_m\chi_n\cdot(-\xi/2)^{(m+n)/2}\,\mathrm{e}^{\mathrm{i}(m-n)\theta}\sqrt{m!n!}}{(\chi_m\cdot m/2)!(\chi_n\cdot n/2)!}(A_{eem}(t)|ee\rangle|m\rangle + A_{+n}(t)|+\rangle|m+1\rangle\\
&+A_{ggm}(t)|gg\rangle|m+2\rangle + B_{ggm}(t)|gg\rangle|m\rangle + B_{+m}(t)|+\rangle|m-1\rangle + B_{eem}(t)|ee\rangle|m-2\rangle)\\
&\times(A_{een}^*(t)|ee\rangle|n\rangle + A_{+n}^*(t)|+\rangle|n+1\rangle + A_{ggn}^*(t)|gg\rangle|n+2\rangle\\
&+B_{ggn}^*(t)|gg\rangle|n\rangle + B_{+n}^*(t)|+\rangle|n-1\rangle + B_{een}^*(t)|ee\rangle|n-2\rangle)\,, \qquad (15)
\end{aligned}$$

with $|\pm\rangle = (1/\sqrt{2})(|e\rangle\pm|g\rangle)$ and

$$\begin{aligned}
A_{een} &= \frac{(n+1)\cos(gt\sqrt{2n+3})+(n+2)}{2n+3}, \quad A_{+n} = \frac{-\mathrm{i}\sqrt{(n+1)(2n+3)}\sin(gt\sqrt{2n+3})}{2n+3},\\
A_{ggn} &= \frac{\sqrt{(n+1)(n+2)}(\cos(gt\sqrt{2n+3})-1)}{2n+3}, \quad B_{ggn} = \frac{n\cos(gt\sqrt{2n-1})+(n-1)}{2n-1},\\
B_{+n} &= \frac{-\mathrm{i}\sqrt{n(2n-1)}\sin(gt\sqrt{2n-1})}{2n-1}, \quad B_{een} = \frac{\sqrt{n(n-1)}(\cos(gt\sqrt{2n-1})-1)}{2n-1}\,. \qquad (16)
\end{aligned}$$

By tracing over the cavity field, the atomic density matrix, in the basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, is again an X-class form (6) with (For the sake of concision, we have ignored the normal parameter $\sqrt{1-\xi^2}$) the elements:

$$\begin{aligned}
a =&\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}(|A_{een}|^2+|B_{een}|^2)+\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^{n+1}\sqrt{n!(n+2)!}}{(\chi_n\cdot n/2)\,!(\chi_{n+2}\cdot(n+2)/2)\,!}(\,\mathrm{e}^{-\mathrm{i}\theta}A_{een}B_{een+2}^*+\,\mathrm{e}^{\mathrm{i}\theta}A_{een}^*B_{een+2}),\\
b =&\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}\frac{(|A_{+,n}|^2+|B_{+,n}|^2)}{2}+\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^{n+1}\sqrt{n!(n+2)!}}{(\chi_n\cdot n/2)\,!(\chi_{n+2}\cdot(n+2)/2)\,!}\Big(\frac{\mathrm{e}^{-\mathrm{i}\theta}A_{+n}B_{+,n+2}^*}{2}+\frac{\mathrm{e}^{-\mathrm{i}\theta}A_{+n}^*B_{+,n+2}}{2}\Big),\\
d =&\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}(|A_{ggn}|^2+|B_{ggn}|^2)+\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^{n+1}\sqrt{n!(n+2)!}}{(\chi_n\cdot n/2)\,!(\chi_{n+2}\cdot(n+2)/2)\,!}(\,\mathrm{e}^{-\mathrm{i}\theta}A_{ggn}B_{ggn+2}^*+\,\mathrm{e}^{\mathrm{i}\theta}A_{ggn}^*B_{ggn+2}),\\
w =&\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^n n!}{[(\chi_n\cdot n/2)\,!]^2}A_{een}B_{ggn}^*+\sum_{n=0}^{\infty}\frac{\chi_n^2(-\xi/2)^{n+1}\sqrt{n!(n+2)!}}{(\chi_n\cdot n/2)\,!(\chi_{n+2}\cdot(n+2)/2)\,!}\,\mathrm{e}^{\mathrm{i}\theta}(A_{ee,n+2}A_{ee,n}^*+B_{ee,n}^*B_{ee,n+2})\\
&+\sum_{n=0}^{\infty}\frac{OE^2(-\xi/2)^{n+2}\sqrt{n!(n+4)!}}{2^{n+2}(OE\cdot n/2)\,!(OE\cdot(n+4)/2)\,!}\times\,\mathrm{e}^{2\mathrm{i}\theta}A_{gg,n}^*B_{ee,n+4},\\
c =&z = b\,. \qquad (17)
\end{aligned}$$

If we substitute these relations to (7), we can easily see that the concurrence is sensitive to the phase θ, which is different from that for the ψ-type Bell state.

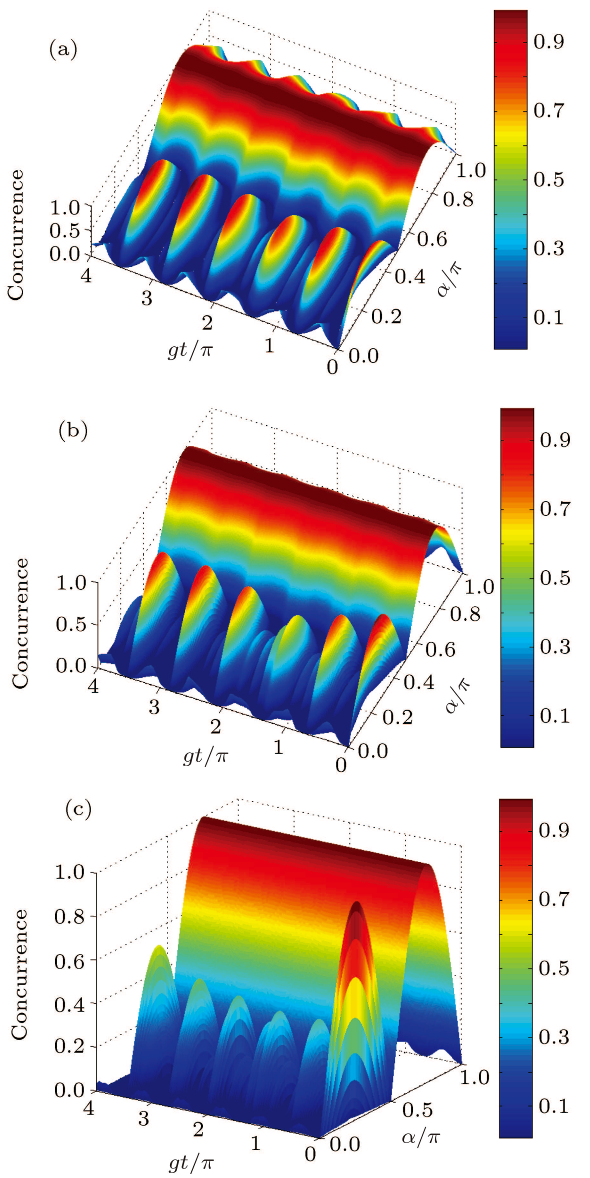

Fig. 2 Time evolution of the concurrence C_{AB} vs. the scale time gt/π and the initial atomic phase α for squeezed vacuum states with (a) $\xi = \sqrt{0.2}$, (b) $\xi = \sqrt{0.5}$, (c) $\xi = \sqrt{0.9}$ for ψ-type initial atomic states.

The time evolution of the concurrence C_{AB} as the function of the scale time gt/π and α is plotted in Fig. 3, where we have set, for simplicity, $\theta = 0$ and $\xi =$ (a) $\sqrt{0.2}$, (b) $\sqrt{0.5}$, (c) $\sqrt{0.9}$. The result shows that the concurrence is also oscillating periodically and can suddenly die in a finite time for $\alpha \in [\pi/2, \pi]$ different from that for ψ-type Bell states with $\alpha \in (0, \pi/2)$. It is also obtained that the larger the mean photon number of SVS is, the more obvious the ESD appears, which is the same to that for ψ-type Bell states. Besides, for the state $|\Phi(\pi/4)\rangle = (|ee\rangle + |gg\rangle)/\sqrt{2}$ the system may not decay as long as ξ is large enough.

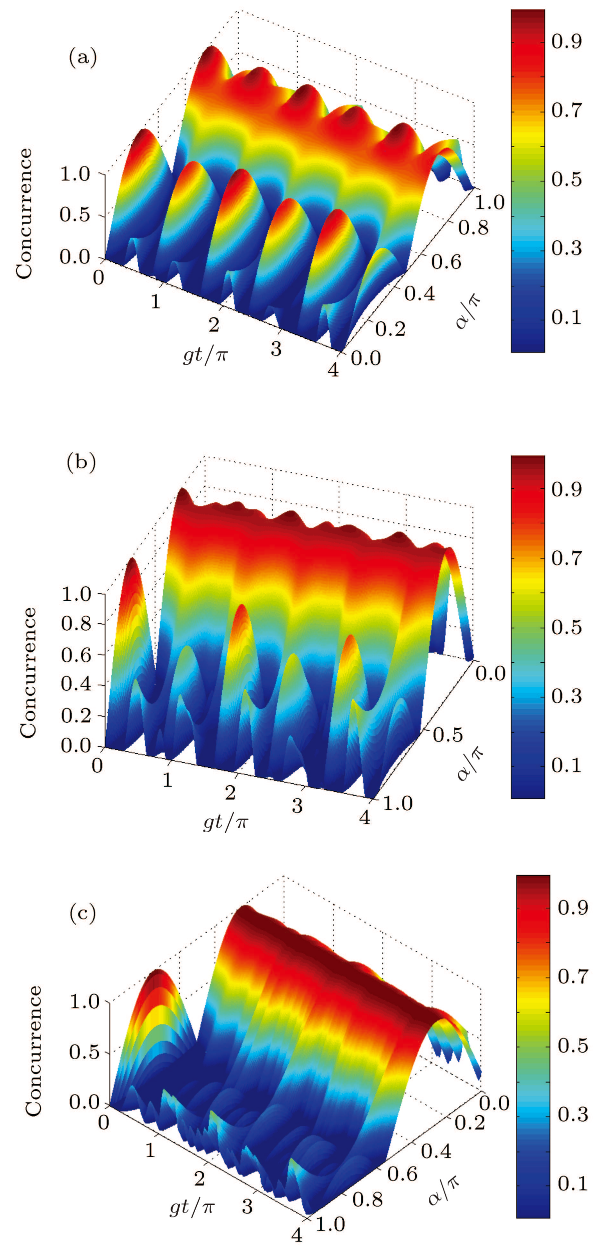

Fig. 3 Time evolution of the concurrence C_{AB} vs. the scale time gt/π and the initial atomic phase α for squeezed vacuum states with $\theta = 0$ and (a) $\xi = \sqrt{0.2}$, (b) $\xi = \sqrt{0.5}$, (c) $\xi = \sqrt{0.9}$ for Φ-type initial atomic states.

We have shown the concurrence versus the scale time gt/π and θ in Fig. 4, where we set $\alpha = 3\pi/4$ and $\xi = \sqrt{0.5}$ without loss of generalization. Our numerical results demonstrate that the concurrence is strongly dependent on the phase θ. The concurrence for $\alpha = 3\pi/4$ and $\theta = \pi$ is similar to that for $\alpha = \pi/4$ and $\theta = 0$. So, we can deduce that the state $|\Phi(\theta)\rangle = (|ee\rangle + \mathrm{e}^{\mathrm{i}\theta}|gg\rangle)/\sqrt{2}$ is a DFS for SVS with the arbitrary phase θ.

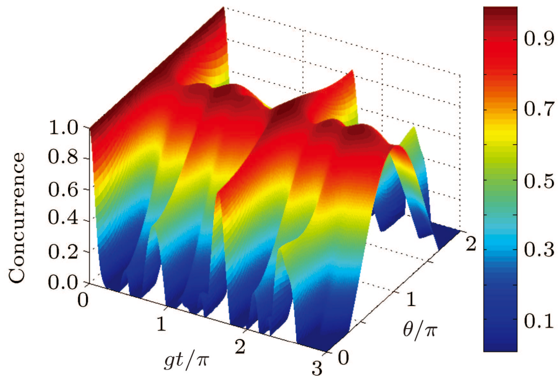

Fig. 4 Time evolution of the concurrence C_{AB} vs θ, with $\xi = \sqrt{0.5}$ and $\alpha = 3\pi/4$.

4 Discussions and Conclusions

Similar to the results obtained from the model for two T–C atoms in the thermal fields, the ESD being considered here for two T–C atoms in the quantum light sources is also related to the initial atomic states and the mean photon number of the field.[30–31] But our result shows that both atoms in the excited states (or in the ground states) can be entangled. They will not be entangled if the two atoms hold only one excited states. We also find that the concurrence for the initial Φ-type Bell states is dependent on the phase of the quantum light sources, which is different from that obtained from the two T–C atoms in the coherent states or in the thermal fields.

Recently, Chen and his collaborators[32] studied the ESD phenomenon for two atoms driven by a strong classical field and they found that ESD can be controlled by the squeezed factor, which is related to the atom-cavity detuning. The smaller the squeezed factor is, the more obvious the ESD demonstrates. In addition, DFS can also be found in the large detuning condition in their scheme. While in our proposal, we find that ESD can be enhanced with the increase of the squeezed factor and the DFS mechanism is quiet different from the results in [23]. Although the nature of the DFS is still not very clear in our scheme, but the distinct feature of the ESD showing here provides some insight into the physics of atoms in a quantum light source.

In conclusion, we have studied the two-atom entanglement based on the Tavis–Cummings atoms interacting with the quantum light sources. Although the physics behind the ESD has not been well understood quantitatively at this moment, we still find that the concurrence is closely related to the initial atomic state and the properties of the light sources. When we confine α within $\alpha \in [0, \pi]$, the concurrence can disappear abruptly in the finite time in the case of $\alpha \in (0, \pi/2)$ for ψ-type Bell states and in the case of $\alpha \in [\pi/2, \pi]$ for Φ-type Bell states. We also show that for large mean photon number of the quantum light sources, corresponding to strong squeezing, the ESD effect is more obvious. In addition, the singlet state $(|eg\rangle - |ge\rangle)/\sqrt{2}$ is always a DFS in the SVS and the entangled state $(|ee\rangle + \mathrm{e}^{\mathrm{i}\theta}|gg\rangle)/\sqrt{2}$ is also a DFS only if the mean photon number of the SVS is large enough. These features are quite different from the previous results obtained by two atoms interacting with the vacuum state or the thermal field. We know that more and more quantum light sources at the transition of the atoms have been generated,[37–38] the system we discuss here could be feasible and the quantum entanglement dynamics can be investigated by using the quantum light sources.

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